The Unconscious Conventionalism of the Physical Geometry of Space

By Matthew Coates

Abstract

Poincaré and Reichenbach argued that the physical geometry of spacetime is conventional: any geometry we wish has the capacity to accurately represent spacetime, so long as it is accompanied by an appropriate physical force field. In this paper, I argue that there also exists an unconscious conventionalism, where we unconsciously choose to dismiss certain possible geometries of spacetime due to incomplete mathematical knowledge, meaning we are unaware of the existence of these other possible choices.

Introduction

With the development of non-Euclidean geometries, the idea of the physical geometry of spacetime being conventional has been championed by philosophers such as Poincaré[106] and Reichenbach[107]. Traditionally this conventionalism has been seen as a conscious choice, due to the empirical character of physical geometry. In this paper I argue there also exists an unconscious conventionalism, due to the mathematical characteristic of physical geometry.

To provide the context for this debate, I will introduce the concept of non-Euclidean geometries, as well as their uses in the development of General Relativity by Einstein. I will also introduce the debate about the possibility of conventionalism in our treatment of the geometry of space. Following from this, I will define my ideas about conscious and unconscious conventionalism in our geometry of space, in order to clarify why they are different.

Whilst I will not make claims about the truth or falsity of conscious conventionalism, I will argue for unconscious conventionalism regarding the geometry of space. I will do this by arguing that there could be gaps in our knowledge, outside of our control, which could still lead to conventionalism in our geometry of space. I will consider one example of this, our mathematical knowledge, and use it to show that it may force on us a conventionalism of our geometry.

My argument will proceed as follows:

(1) If we choose to dismiss the possibility of the geometry of space being certain geometries, without realising we have made such a choice, then the geometry of space is unconsciously conventional.

---

106 Poincaré, Henri. *Science and Hypothesis.* (New York: The Walter Scott Publishing Co. Ltd, 1905) p.58

107 Reichenbach, Hans. The Philosophy of Space and Time. (New York: Dover Publications, 1957) pp. 10-14

(2) If we do not have complete knowledge of mathematics, we may not know of the mathematical existence of certain geometries.
(3) If we do not know of the mathematical existence of certain geometries, we may choose to dismiss the possibility of the geometry of space being certain geometries, without realising we have made such a choice.
(4) We do not have complete knowledge of mathematics.
(5) We may not know of the mathematical existence of certain geometries
(6) We may choose to dismiss the possibility of the geometry of space being certain geometries, without realising we have made such a choice.
(C) The geometry of space is unconsciously conventional.

Because of the geometry of space being unconsciously conventional, it then follows that we cannot know that the geometry of spacetime is a certain form on non-Euclidean geometry. Following this I will discuss two problems that this claim could possibly face, providing a solution to one, and ideas for a potential solution to the other.

Non-Euclidean Geometry and Convention

One of the biggest mathematical developments in the 19th century was the discovery of non-Euclidean geometries, by mathematicians such as Gauss, Lobachevsky and Bolyai. These are geometries in which not all the axioms of geometry laid out by Euclid hold. One way of creating such a non-Euclidean geometry is through the replacement of the parallel postulate of Euclid by another axiom. The parallel postulate states:

P: Given a line, there exists one straight line through any point which is parallel to the given line.

We can then see two potential alternatives to this parallel postulate:

P_1: Given a line, there exists more than one straight line through a point parallel to the given line (hyperbolic geometry),

P_2: Given a line, there exists no straight line through a point parallel to the given line (elliptic geometry).

Despite the removal of the parallel axiom, these new geometries remain mutually consistent with Euclidean geometry. Hilbert showed that if a contradiction was found in one of these non-Euclidean geometries, then a contradiction could also be found in Euclidean geometry. Hence, if Euclidean geometry is consistent, then so are hyperbolic and elliptic geometry. For this reason, it appears that we are equally justified in believing in these new geometries as we are in Euclidean geometry. Euclidean geometry, and its connection with the geometry of space, has often been afforded a special status in the realm of philosophy. It was held by Kant, writing before the discovery of different geometries, that Euclidean geometry, including the parallel axiom, was an a priori truth that described space[108]. With the development of these non-Euclidean geometries, this position became difficult to hold. Why should Euclidean geometry hold a special status, when equally consistent yet different geometries also existed, especially as the geometry on a sphere seems to adhere to elliptic geometry?

The position held by Kant became even more difficult after the formulation of General Relativity. General Relativity posits that spacetime is curved in the presence of matter and energy. This curvature is associated with deviations from the structure of Euclidean geometry. Whilst this deviation is so small for relatively small matter-energy densities, leading us to believe that space is Euclidean, this effect increases in the presence of larger bodies. Due to the discovery of this, it no longer appeared that space adheres to the parallel axiom that Kant held to be a priori.

The existence of these different geometries led Poincare to argue[109] that what we perceive to be the physical geometry of space is just a convention. We cannot truly know the physical geometry of space, and instead choose it from a group of possible candidates. Poincaré argued that physical geometry is just a reflection of the properties of our measuring equipment, and hence can be chosen conventionally[110]. This was demonstrated by the following example. Imagine a disk, which is incredibly hot in the centre and cooling to absolute zero as you reach the circumference. Any measuring equipment placed on this disk immediately takes the temperature on the disk. As materials contract when the temperature gets lower, there are two clear effects that this would cause. The first is that the disk, when measured, would have an infinitely long diameter, as the measuring rods would contract to no size when they reach absolute zero. The second effect is that this heat causes measurements of the geodesics of this geometry to appear hyperbolic non-Euclidean when measured with equipment that is heat resistant, however when measured with heat resistant equipment it appears Euclidean. Poincaré's argument from this is that physical geometry is conventional, based on what we choose to measure with, because measuring with different materials would make us perceive space to hold different physical geometries. Different forces can be used, instead of that of heat as in Poincaré's example, which would affect geometry in different ways.

Conscious and Unconscious Conventionalism

---

[108] Shapiro, Stewart. *Thinking about Mathematics: The Philosophy of Mathematics*. (Oxford: Oxford University Press, 2000) p.89

[109] Poincaré, Henri. *Science and Hypothesis.* (New York: The Walter Scott Publishing Co. Ltd, 1905) p.58

[110] ibid

I will now introduce the concepts of conscious and unconscious conventionalism. Conscious conventionalism is the traditional view of conventionalism as given in many forms by philosophers such as Poincaré[111] or Reichenbach[112]. This view claims that the choice of physical geometry is conventional, meaning that it can be chosen by whoever is attempting to model the physical geometry of space. We can do this by positing some forces, such as heat in Poincaré's example, which makes us perceive our spacetimes geometry as some different geometry.

This is demonstrated by the previous example of Poincaré's disk. In this example, we have a spacetime that is Euclidean, unknown to those living on it. However, we have posited a force, in this case heat, which, due to its effect on measuring equipment, makes us measure the geometry of our spacetime as being hyperbolic. This example demonstrates that whilst we may consider our geometry to be non-Euclidean, it could be the result of some force on a Euclidean geometry making us perceive it as being non-Euclidean.

As demonstrated by this example, we have made a conscious decision about what geometry we are using. We make a conscious decision that our geometry is non-Euclidean. We would be equally justified in making a conscious decision to say our geometry is Euclidean, appearing non-Euclidean because of a force.

Unconscious conventionalism however, is distinguished from conscious conventionalism by the idea that we could have to make a conventional choice about which geometry we use to describe space, without knowing we are actually making such a choice. It would be an unconscious choice. How would this happen? We could have incomplete information about everything that is being modelled, such as the mathematics needed. We would then not recognise the possible existence of alternative models. When choosing a model, we may hence dismiss the possibility of certain models being correct. This would be unconscious because we do not realise they could mathematically exist and would not even knowingly consider them.

An example of this unconscious conventionalism is attempting to model the structure of non-Euclidean space without knowing that non-Euclidean geometries are mathematically possible. In this case, we make an unconscious choice to dismiss the idea that space could be non-Euclidean, because we do not know of the mathematical existence of non-Euclidean geometries. We then choose a geometry out of the remaining ones that we do know exist, which in this case is only that space is Euclidean.

---

111 ibid

112 Reichenbach, Hans. The Philosophy of Space and Time. (New York: Dover Publications, 1957) pp. 10-14

Before arguing for this position, I will comment on the name "unconscious conventionalism." The example of Euclidean geometry mentioned previously has called into question what exactly is conventional about "unconscious conventionalism." Surely, if we are forced to choose Euclidean geometry, because we do not know of non-Euclidean geometries, that is the opposite of conventionalism? This is because we have no choice in what we choose. However, I believe that this still satisfies the term conventionalism. This is because we make a choice about which categories of geometries to dismiss, without realising we are making such a choice, based on our current mathematical knowledge. Unconscious conventionalism is a concept based around unconsciously making a choice of what to dismiss.

For Unconscious Conventionalism

Physical geometry has, at various times, been considered to be either a mathematical or an empirical discipline. Traditionally, especially prior to the development of non-Euclidean geometries, physical geometry was considered to be wholly mathematical, for example by Kant. Kant considered the geometric structure of space to be an a priori truth. This truth can be known through our mathematical investigations into geometry. However, there also exists an empirical discipline of physical geometry, as believed by philosophers such as Carnap[113]. In this empirical geometry, we can only come to know the physical geometry of space through empirical theories and observations. Instead of this separated view however, I will claim that physical geometry must be considered a combination of the mathematical and empirical. This is because, whilst we can only come to gain knowledge of physical geometry through empirical observation, we have to describe the relations between those observations using a mathematical framework. This leads to the idea of unconscious conventionalism. If our mathematical knowledge is incomplete, we may unconsciously dismiss that the geometric structure of spacetime can be certain types of geometry, because we do not know they can mathematically exist.

I will make the assumption that never, in the history of mathematics, has mathematical knowledge been changed due to some changes in the natural sciences[114], and is hence autonomous. Whilst mathematics has been changed or corrected, it has always been due to mathematics itself, not the natural sciences. This is demonstrated by the discovery of non-Euclidean geometries. This was a mathematical discovery, with no perceived connection to the physical world. It was only following the

---

113 Carnap, Rudolf. An Introduction to the Philosophy of Science. (New York: Dover Publications, 1966/1995) p.168

114 Brown, James R. Philosophy of Mathematics. (London: Routledge, 2000) p.56

mathematical discovery of these non-Euclidean geometries that it could then be used to model the physical world.

A distinction should however, be made between the natural sciences changing mathematics, and the natural sciences inspiring mathematics. Whilst mathematics has never changed due to developments in the natural sciences, the natural sciences have inspired further work in mathematics. An example of this would be General Relativity stimulating developments in differential geometry. I am focusing on mathematics changing, as it requires development outside of the framework of current mathematical knowledge, instead of just advancing development in areas already known.
I will similarly assume that mathematical knowledge is not complete. This means that we do not know everything there is to know in mathematics. There are two reasons to believe this claim. The first is that mathematical research is still taking place, and new mathematical knowledge is being discovered. The second is through examples from the history of mathematics, such as the development of non-Euclidean geometry, or that of Set Theory.
There are many cases where these increases in mathematical knowledge would go on to, in the future, influence the development of theories in the sciences that would not have been feasible without them. The obvious example is that of the transition between Euclidean and non-Euclidean geometry, allowing new ways of modelling the physical world, and leading to the development of General Relativity. If the possible mathematical existence of non-Euclidean geometry had not been discovered, it follows from the first assumption, that mathematics is autonomous from the natural sciences, and that any theory formulated to account for the same effects as General Relativity would have unconsciously chosen to dismiss the possibility of non-Euclidean geometry from the selection of possible geometries. This is because, in this case, non-Euclidean geometries would not be known to be mathematically possible, and therefore would not be considered when modelling the physical geometry of space.
Because of this, physicists would have had to justify their seemingly non-Euclidean measurements with various forces, in order to account for the differences. The structure of the physical geometry of space would then be considered to be Euclidean, with forces changing the structure of it in some way. The Euclidean geometry would have been chosen unconsciously, because it is assumed that the physicists would have no idea that non-Euclidean geometries could exist, and hence not know of the existence of other possible choices. Due to this, they would naturally try to account for it in ways that are considered to be true, such as the existence of more forces, rather than ways that are not.

As a result of these arguments, it follows that we cannot say that the geometry of spacetime is a certain form of non-Euclidean geometry. We cannot be sure that we have enough mathematical knowledge to describe the geometry of spacetime fully. Instead we choose from a selection of known geometries to describe the geometry of spacetime, unconsciously dismissing any types of geometries that we are unaware of, but may mathematically exist. We hence unconsciously choose that the geometry of spacetime cannot take the form of these geometries which we do not know the mathematical existence of, dismissing them and choosing one which we do know exists[115]. We then must add forces which alter our chosen geometry, to match the mathematical representation with the empirical observation. These forces can be chosen in a consciously conventional way.

This working hand in hand of unconscious conventionalism and conscious conventionalism can be demonstrated by Newton's development of his theory of gravitation. Unlike Einstein, who knew of the possible existence of non-Euclidean geometries, Newton unconsciously dismissed their existence. Unconsciously dismissing those non-Euclidean geometries, he chose the only type of geometry he knew, Euclidean geometry, to model his approach to gravity. He then had to consciously conventionally posit forces in order to save the phenomena.

Problem

There does however appear to be a problem with this account. The possible criticism is that the physicist, working on a theory such as General Relativity, may recognise that the observations do not match up with Euclidean geometry. They may then assume that the physical geometry of spacetime is some form of non-Euclidean geometry, not unconsciously dismissing the possibility of that non-Euclidean geometry. This is despite having no proven mathematical knowledge that such non-Euclidean geometries could exist. In doing so, it seems they would no longer need to account for the difference between mathematical knowledge and perceived reality.

This possible objection however, instead justifies the conventionalism of unconscious conventionalism. Whilst this physicist may have not chosen to reject that specific non-Euclidean geometry, they have still made the unconscious choice to dismiss a large number of other possible geometries. They unconsciously choose to dismiss all other non-Euclidean geometries, leaving them with Euclidean geometry and their non-Euclidean geometry to choose from. This is in comparison to somebody who has unconsciously dismissed all but Euclidean

---

[115] This further choice between geometries that we do know exist can be consciously convention.

geometry. Hence there is an unconscious choice about which are rejected, instead of a forced rejection of all. This choice in dismissal is still without knowing that they are dismissing all of these other possible geometries that they do not know could exist and so would still be unconscious.

The choice they would have, following this, could then involve choosing between the geometries which they have not dismissed. This choice could either have one correct answer, the non-Euclidean geometry they had noticed, or be consciously conventional and have either the non-Euclidean geometry or the Euclidean geometry be correct. This would not be important to truth or falsity of unconscious conventionalism however.

Another problem is that such a discovery, for example of the geometry of space appearing non-Euclidean, could have forced mathematicians to begin developing non-Euclidean geometry. I believe that the answer to this is that it is not really a problem. The physicist may be unconsciously conventional about their geometry whilst not knowing that non-Euclidean geometry can exist, building their theories around this. At the same time, this could lead mathematicians or physicists to attempt to prove that non-Euclidean geometry can exist, leading to the possible updating of theories about the physical geometry of space, removing the conventional forces and making it non-Euclidean. This new theory could then be possibly unconsciously conventional due to some other piece of mathematics we do not know yet know.

Conclusion

In conclusion, I believe that I have shown that there is a possibility that we are unconsciously conventional about our choices of representation of physical geometry. Due to the mathematical nature of physical geometry, as well as our potentially incomplete knowledge of mathematics, we may unconsciously choose to dismiss the possibility of the physical geometry of space being certain geometric structures, based on our current of mathematics.

Bibliography

Brown, James Robert. 2000. *Philosophy of Mathematics*. London: Routledge.

Carnap, Rudolf. 1995/1966. *An Introduction to the Philosophy of Science.* New York : Dover Publications.

Poincaré, Henri. 1905. *Science and Hypothesis*. New York: The Walter Scott Publishing Co. Ltd.

Reichenbach, Hans. 1957. *The Philosophy of Space and Time.* New York: Dover Publications.

Shapiro, Stewart. 2000. *Thinking about Mathematics: The Philosophy of Mathematics.* Oxford: Oxford University Press.